

Name: _____

MTH 330 Final Review

1. Calculate the l_2 and l_∞ norms of the following matrix: $A = \begin{bmatrix} 5 & -1 \\ 2 & 2 \end{bmatrix}$

2. Find an upper triangular matrix U and a lower triangular matrix L such that $LU = A$ where

$$A = \begin{bmatrix} 1 & 3 & -5 \\ 3 & 7 & 6 \\ 2 & 0 & 2 \end{bmatrix}$$

3. Let $\mathbf{b} = [1 \ 24 \ 14]^T$ and use your answer to #2 to solve $A\mathbf{x} = \mathbf{b}$.

4a. Using the l_∞ norm, compute the condition number $\kappa(A)$ of the following matrix:

$$A = \begin{bmatrix} 3 & -5 \\ 1 & 7 \end{bmatrix}$$

b. Let $\mathbf{b} = [1 \ 9]^T$, and let $\tilde{\mathbf{x}} = [1 \ 1]$ be an approximate solution to the system $A\mathbf{x} = \mathbf{b}$. Compute upper and lower bounds on $\|\mathbf{e}\|$ and $\|\mathbf{e}\|/\|\mathbf{r}\|$ where $\mathbf{e} = \tilde{\mathbf{x}} - \mathbf{x}$ and $\mathbf{r} = A\tilde{\mathbf{x}} - \mathbf{b}$.

6. Perform Gaussian elimination on the following matrix using Scaled Partial Pivoting:

$$A = \begin{bmatrix} -2 & 9 & 0 \\ 7 & 1 & 10 \\ 4 & 2 & 2 \end{bmatrix}$$

7. Find a quadratic polynomial $P(x)$ that best fits the points $(0, 1)$, $(1, 4)$, and $(2, 9)$ using Lagrange Polynomials.

8. Use Neville's Algorithm to approximate $f(1.3)$ if you know that $f(0) = 2$, $f(1) = -1$, and $f(2) = 4$.